2.4 The Chain Rule

Exploration/warmup

1) Use the basic power rule to find $f'(x)$ given $f(x)=(x+3)^2$. Next, multiply out and then differentiate. Do you notice any similarities or differences?

2) Now, use the basic power rule to find $g'(x)$ given $g(x)=(2x-1)^2$. Next, multiply out and then differentiate. Do you notice any similarities or differences?

3) Next, use the basic power rule to find $h'(x)$ given $h(x)=(3x^2+1)^2$. Next, multiply out and then differentiate. Do you notice any similarities or differences?

4) Next, use the basic power rule to find $m'(x)$ given $m(x)=(x^3+x)^2$. Next, multiply out and then differentiate. Do you notice any similarities or differences?

Composite functions:
if f(x) = x^2

and g(x)= (2x+3) what is f(g(x))

What if f(x)=x^{10}

When differentiating functions with inside parts and outside parts, we must use **The Chain Rule so we don't have to multiply 10 times**.

The Chain Rule

If $y = f(u)$ is a differentiable function of *u* and $u = g(x)$ is a differentiable function of *x*, then $y = f(g(x))$ is a differentiable function of *x* and

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x) \qquad \text{or} \qquad y' = f'(u)\cdot u' \qquad \text{or}$$

Ex. 1: Use the Chain Rule to find the derivative:

Let f(x)=$(2x+3)^2$, Find f'(x).

Practice:

a) $f(x) = (3x - 2x^2)^3$

b) $y = \sqrt{x^2 + 1}$

c) $g(t) = \dfrac{-7}{(2t-3)^2}$

d) $f(x) = (5 - x)^2(3x + 1)^3$

e) y=cos($3x^3$)

f) y=sec(tanx)

<u>The General Power Rule</u>

If $y=\left[u(x)\right]^n$, where *u* is a differentiable function of *x* and *n* is a rational number, then
$\frac{d}{dx}\left[u^n\right]=nu^{n-1}u'$

<u>Ex. 2:</u> Find all points on the graph of $f(x)=\sqrt[3]{\left(x^2-1\right)^2}$ for which $f'(x)=0$ and those for which $f'(x)$ does not exist. Then graph to verify your answers.

<u>Ex 3</u>: An object moves along the x axis so that its position at any time is given by $x(t)=\cos(t^2+1)$ for $t\geq 0$. Find the velocity as a function of time.

<u>Ex 4</u>: Suppose that h(x)=f(g(x)) and that f '(3)=2, f(3)= 4, g(5)=3, g'(3)= 1 and g'(5)=7. Find h'(5).

Practice: Let *f* and *g* be differentiable functions such that *h(x)=f(g(x)),*

f(1)=4	*g(1)=3*	*f ' (3)=-5*	
f '(1)=-4	*g ' (1)= -3*	*g '(3)=2*	*Find h'(1)*

<u>Ex. 5:</u> More complicated. Find the derivative of each of the following:

a) $f(x)=\dfrac{x}{\sqrt[3]{x^2+4}}$

b) $y=\left(\dfrac{3x-1}{x^2+3}\right)^2$

Nested functions…the chain rule more than once (often with trig)

Ex. 6: $f(\theta)=2\sin^2(2\theta)$

Try: $f(\theta)=\cos^2(3\theta)$

$y=\tan(\sin(x^2))$

$f(t)=\sin^3 4t$

$y=\sqrt{\csc(x^3+5x)}$